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The influence of vortex shedding on the diffraction of sound by a perforated screen

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This paper examines the theory of the interaction of sound with a slit-perforated screen in the presence of a uniform, subsonic tangential mean flow on both sides of the screen. The sound induces vortex shedding from sharp edges of the screen. The coupling of this vorticity with the mean flow leads to a significant modification in the predicted acoustic properties as compared with those predicted by the classical treatments of Rayleigh (1897) and Lamb (1932). In particular a considerable portion of the incident acoustic energy can be lost during the interaction, and is convected away in the mean flow in the form of localized vortical disturbances. The analytical results provide theoretical support for the use of perforated plates to inhibit the onset of cavity resonances in, for example, cross-flow heat exchangers.

1. Introduction

Lamb (1932, § 306) has discussed the diffraction of a plane sound wave by a thin screen which is perforated by a series of parallel, equal, and equidistant slits. The case of a screen perforated with small circular apertures was examined by Ffowcs Williams (1972), and in more detail for the general elliptic aperture by Leppington & Levine (1973). The effect of a low-Mach-number 'bias' flow (i.e., of a mean normal flow through the screen) has been considered for a screen with circular apertures by Howe (1979*a*). In this case there exists a mean jet-like flow in the wake of each aperture whose structure is modulated by the incident sound through the unsteady shedding of vorticity from the rim of the aperture. In this paper we examine the diffraction of sound when the screen is located in a uniform *tangential* mean flow. In contrast to the bias flow problem, any shed vorticity remains in the vicinity of the screen, so that in the absence of viscous dissipation, its influence on the unsteady motion in the apertures can persist indefinitely.

The analysis will proceed from the linearized equations of motion and will deal with the slit-perforated screen studied by Lamb, the tangential flow being at right angles to the slits. A purely acoustic (irrotational) treatment of this problem would involve the appearance of infinite velocity and pressure fluctuations at the edges of the slits. The singular behaviour can be removed at the upstream edges A (see figure 1) by applying the Kutta condition, and this leads to the shedding of vorticity which is swept along the screen by the mean flow. On linear theory, vorticity which is shed from the leading edges of the rigid portions of the screen is immediately annulled by image vortices in the screen. This precludes an application of the Kutta condition at such points where, as in thin airfoil theory (cf. Ashley & Landahl 1965, § 13.2), large

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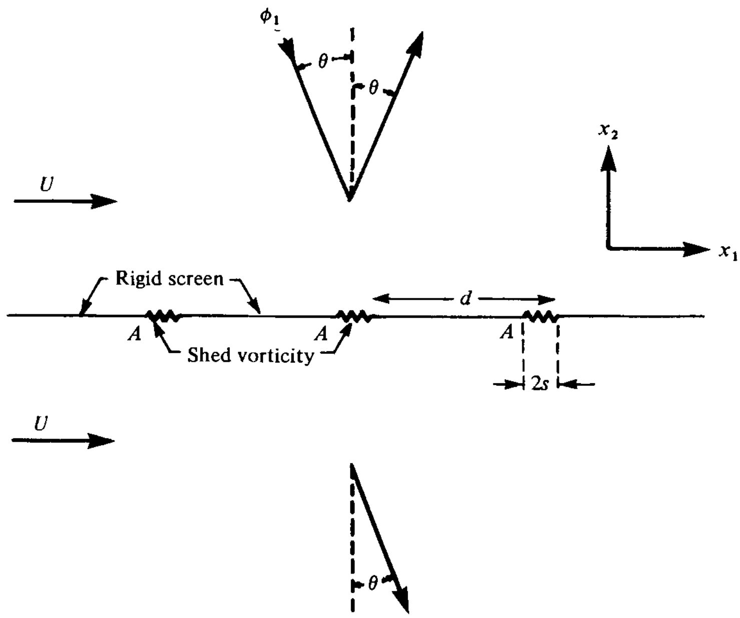

FIGURE 1. Schematic illustration of the perforated screen. A plane acoustic wave impinges on the screen from $x_2 > 0$ and induces the shedding of vorticity from the upstream edges A of the slits.

fluctuations in pressure and velocity can develop. The unsteady vorticity gives rise to a time dependent potential difference across the screen, which influences the volume flux and causes acoustic energy to be dissipated. The magnitude of this potential difference is governed by interactions of the vorticity with the edges of the slits, and is accordingly dependent on the Strouhal number based on the slit width and the vorticity convection velocity. For certain discrete frequencies these interactions produce no net potential difference, and the diffraction of sound is then essentially unaffected by the mean flow.

The dissipation of acoustic energy by a perforated plate in a tangential flow has important applications, for example, to the problem of controlling acoustic resonances in heat exchangers in gas-cooled nuclear reactors. Vortex shedding from tube-banks in the heat exchanger cavity generates aerodynamic sound (Lighthill 1952; Walker & Reising 1968) which can be resonantly coupled to the vibrations of tubes, plates and other structural members. The resulting oscillating stresses (which might well correspond to sound pressure levels of 150 dB, re 0·002 μbar, or more) can lead to metal fatigue and structural failure. Various devices have been proposed for suppressing these resonances, including the use of baffles and the complete removal of several judiciously placed tubes to inhibit 'in-phase' oscillations. Vér (1979, private communication), however, has recently demonstrated that the insertion of one or more perforated plates parallel to the mean flow is a particularly effective means of eliminating the resonances.

The diffraction problem is formulated in terms of an integral equation in § 2 of this paper. In § 3 it is assumed that the spacing of the slits is small compared with the relevant Lorentz contracted acoustic wavelength, but large compared with the width of each slit. This permits the derivation of an analytic solution, and is also the appropriate limit for applications. The dissipation of acoustic energy is discussed in §4, where comparison is made with Lamb's (1932) solution, and with nonlinear dissipative mechanisms in the slit (Ingard & Ising 1967).

2. Formulation of the diffraction problem

Consider the problem illustrated schematically in figure 1, in which a plane sound wave of angular frequency $\omega > 0$ propagates in an ideal fluid. The wave is incident on a thin, rigid screen which occupies the plane $x_2 = 0$ of a rectangular co-ordinate system (x_1, x_2, x_3) and is perforated with an array of parallel, equal, and equidistant slits. Let $2s$ denote the slit width, and let d be the distance between the centre-lines of adjacent slits. The origin of co-ordinates is taken in the centre of one of the slits with the x_3 axis parallel to the slits and directed out of the paper in figure 1.

There is a uniform mean flow at speed U parallel to the positive direction of the x_1 axis, so that if $\Phi\, e^{-i\omega t}$ denotes the potential of time-harmonic acoustic perturbations, Φ satisfies the convected form of the wave equation:

$$\left\{\left(-ik+M\frac{\partial}{\partial x_1}\right)^2-\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}\right)\right\}\Phi = 0, \tag{2.1}$$

where $k = \omega/c$, c is the speed of sound, and $M = U/c$ is the Mach number of the mean flow. We confine attention to the subsonic case $M < 1$.

Let $\Phi_I e^{-i\omega t}$ represent the potential of the incident wave which is assumed to impinge on the screen from $x_2 > 0$. It is required to determine the reflected and transmitted fields when account is taken of the possibility of vortex shedding from the upstream edges A of the slits. The strength of the shed vorticity will be chosen in accordance with the Kutta condition that the fluctuating pressure and velocity are finite at A. In the absence of viscous diffusion, the vorticity will occupy a thin sheet in the plane $x_2 = 0$ of the screen. The presence of this unsteady vortex sheet implies that the potential Φ is discontinuous across $x_2 = 0$ in each of the slits. The perturbation pressure $p\, e^{-i\omega t}$, say, must be continuous across the slits, however, and it follows from Bernoulli's equation

$$p = -\rho_0\left(-i\omega+U\frac{\partial}{\partial x_1}\right)\Phi, \tag{2.2}$$

where ρ_0 is the mean density, that the discontinuity in Φ can be represented in the form

$$\begin{aligned}[\Phi] &\equiv \Phi(x_1, +0, x_3)-\Phi(x_1, -0, x_3)\\ &= \alpha\, e^{i\kappa x_1},\end{aligned} \tag{2.3}$$

where α is a constant for each slit, and $\kappa = \omega/U$ is the hydrodynamic wavenumber. Equation (2.3) is equivalent to the result that, on linear theory, shed vorticity convects passively at the mean stream velocity U. The interpretation of formulae derived below is facilitated by the introduction of a subscript c to characterize the motion of the vorticity, i.e., we take the vorticity convection velocity to be U_c. This enables the influence of the vorticity to be followed through the analysis, and to be distinguished from other effects of the mean flow. In this case the hydrodynamic wavenumber is defined by

$$\kappa = \omega/U_c. \tag{2.4}$$

The constant α in (2.3) is fixed by the Kutta condition. The remaining conditions to be satisfied are that $\partial\Phi/\partial x_2 = 0$ on the rigid portions of the screen and, since the mean flow is the same on both sides of the screen, is continuous across each of the slits.

If the Prandtl–Glauert transformations

$$\Phi = \psi\, e^{-ikMx_1/(1-M^2)}, \tag{2.5}$$

$$(X_1, X_2, X_3) = (x_1/(1-M^2)^{\frac{1}{2}}, x_2, x_3) \tag{2.6}$$

and

$$K = k/(1-M^2)^{\frac{1}{2}} \tag{2.7}$$

are introduced into equation (2.1), the potential ψ is found to satisfy the homogeneous Helmholtz equation

$$\left(\frac{\partial^2}{\partial X_1^2}+\frac{\partial^2}{\partial X_2^2}+\frac{\partial^2}{\partial X_3^2}+K^2\right)\psi = 0, \tag{2.8}$$

in $X_2 \gtrless 0$, where $\partial\psi/\partial X_2 = 0$ on the rigid portions of the screen and is continuous elsewhere. The jump condition (2.3) becomes

$$[\psi] = \alpha\, e^{i(\bar{\kappa}+KM)X_1}, \tag{2.9}$$

where

$$\bar{\kappa} = \kappa(1-M^2)^{\frac{1}{2}}. \tag{2.10}$$

Let the incident plane wave Φ_I have the Prandtl–Glauert representation

$$\psi_I = e^{i(n_1 X_1 - n_2 X_2 + n_3 X_3)} \quad (n_2 > 0). \tag{2.11}$$

Conditions are homogeneous with respect to X_3 so that all components of the acoustic field will be proportional to $e^{in_3 X_3}$, and we may temporarily suppress the explicit dependence on this factor. Thus, writing

$$\Gamma = (K^2 - n_3^2)^{\frac{1}{2}}, \tag{2.12}$$

the branch-cuts being chosen such that $\Gamma \to K$ as $|K| \to \infty$, equation (2.8) becomes

$$\left(\frac{\partial^2}{\partial X_1^2}+\frac{\partial^2}{\partial X_2^2}+\Gamma^2\right)\psi = 0. \tag{2.13}$$

In $X_2 > 0$ we set

$$\psi = e^{i(n_1 X_1 - n_2 X_2)} + e^{i(n_1 X_1 + n_2 X_2)} + \psi_s(X_1, X_2), \tag{2.14}$$

where the second term on the right is the field that would be reflected from the screen in the absence of perforations, and ψ_s is the contribution from the interaction with the slits. Note that $\partial\psi_s/\partial X_2$ vanishes on the screen.

The boundary-value problem for ψ may be reduced to an integral equation for

$$V(X_1) \equiv \partial\psi_s(X_1, 0)/\partial X_2 \tag{2.15}$$

which is non-zero only in the slits (cf. Leppington & Levine 1973). To do this we make use of the Green's function

$$G(\mathbf{r},\mathbf{r}') = -\tfrac{1}{4}i\{H_0^{(1)}(\Gamma|\mathbf{r}-\mathbf{r}'|)+H_0^{(1)}(\Gamma|\bar{\mathbf{r}}-\mathbf{r}'|)\}, \tag{2.16}$$

defined in terms of the Hankel function $H_0^{(1)}(z)$, where

$$\mathbf{r} = (X_1, X_2); \quad \bar{\mathbf{r}} = (X_1, -X_2); \quad \mathbf{r}' = (Y_1, Y_2). \tag{2.17}$$

Application of Green's formula to $\psi_s(\mathbf{r}')$ and $G(\mathbf{r},\mathbf{r}')$ then shows that in $X_2 > 0$,

$$\psi_s(\mathbf{r}) = \sum_{N=-\infty}^{\infty}\int_{S_N} G(X_1, X_2;\ Y_1, 0)\, V(Y_1)\, dY_1, \tag{2.18}$$

the integration being over all of the slits, S_N denoting the slit whose centre-line is at $X_1 = N\bar{d}$, where $\bar{d} = d/(1-M^2)^{\frac{1}{2}}$. Similarly in $X_2 < 0$ the potential $\psi(\mathbf{r})$ is given by

$$\psi(\mathbf{r}) = -\sum_{N=-\infty}^{\infty}\int_{S_N} G(X_1, X_2;\ Y_1, 0)\, V(Y_1)\, dY_1. \tag{2.19}$$

The periodicity of the incident wave and of the spacing of the slits suggests that we seek a solution which satisfies

$$\psi_s(X_1+N\bar{d}, X_2) = \psi_s(X_1, X_2)\, e^{in_1 N\bar{d}}, \tag{2.20}$$

in which case (2.14), (2.18) imply that in $X_2 > 0$:

$$\psi(X_1, X_2) = e^{i(n_1X_1-n_2X_2)} + e^{i(n_1X_1+n_2X_2)} + \int_{-\bar{s}}^{\bar{s}} \mathscr{G}(X_1, X_2;\ Y_1)\, V(Y_1)\, dY_1, \tag{2.21}$$

where

$$\mathscr{G}(X_1, X_2;\ Y_1) = \sum_{N=-\infty}^{\infty} G(X_1, X_2;\ Y_1+N\bar{d}, 0)\, e^{in_1N\bar{d}}, \tag{2.22}$$

and $\bar{s} = s/(1-M^2)^{\frac{1}{2}}$. Likewise we have in $X_2 < 0$, from (2.19),

$$\psi(X_1, X_2) = -\int_{-\bar{s}}^{\bar{s}} \mathscr{G}(X_1, X_2;\ Y_1)\, V(Y_1)\, dY_1. \tag{2.23}$$

The substitution of (2.21), (2.23) into the jump condition (2.9) across the slit S_0 which contains the origin yields the following equation for $V(X_1)$:

$$2\, e^{in_1X_1} + 2\int_{-\bar{s}}^{\bar{s}} \mathscr{G}(X_1, 0;\ Y_1)\, V(Y_1)\, dY_1 = \alpha\, e^{i(\bar{\kappa}+KM)X_1} \quad (|X_1| < \bar{s}). \tag{2.24}$$

The value of the constant α in this equation is to be calculated from the condition that

$$V(X_1) \equiv \exp\,[ikMx_1/(1-M^2)^{\frac{1}{2}}]\, \partial\Phi/\partial x_2$$

must remain finite as $X_1 \to -\bar{s}$, i.e., at the upstream edge A of S_0. When $V(X_1)$ has been determined from (2.24), the acoustic fields in $X_2 \gtrless 0$ will be given respectively by (2.21), (2.23).

3. The diffracted field at low Helmholtz number

In order to solve equation (2.24) in analytic form it is necessary to simplify the kernel function of the integral. To do this we introduce the hypothesis

$$2\Gamma\bar{s} \ll \Gamma\bar{d} \ll 1. \tag{3.1}$$

This is equivalent to requiring that

$$\frac{kd}{1-M^2} \ll 1, \tag{3.2}$$

where $kd = \omega d/c$ is the *Helmholtz number*. The analysis is accordingly restricted in validity to wavelengths which are large compared with the spacing of the slits, and to relatively low subsonic mean flow Mach numbers. Both of these conditions are relevant in applications, and permit the summation in (2.22) to be evaluated asymptotically for small $\Gamma\bar{d}$ (c.f., the analogous problem treated by Leppington & Levine 1973).

Using results tabulated by Gradshteyn & Ryzhik (1965, p. 976) we find that

$$\sum_{N=-\infty}^{\infty} G(X_1, 0; Y_1+N\bar{d}, 0)\, e^{in_1 N\bar{d}} = \frac{1}{\pi}\ln\left(\frac{2\pi|X_1-Y_1|}{\bar{d}}\right) - \frac{i}{n_2\bar{d}} + \ldots, \tag{3.3}$$

where the terms not shown explicitly vanish as $\Gamma\bar{d} \to 0$. When this and the dimensionless variables

$$\xi = X_1/\bar{s}, \quad \mu = Y_1/\bar{s} \tag{3.4}$$

are introduced into equation (2.24) we obtain:

$$\int_{-1}^{1} V(\mu)\ln|\xi-\mu|\, d\mu = g(\xi) \quad (|\xi| < 1), \tag{3.5}$$

where

$$\left.\begin{aligned} g(\xi) &= \frac{\pi}{\bar{s}}\left\{\frac{\alpha}{2}e^{i\kappa s\xi} - (1+C\bar{s}Q)\right\}, \\ Q &= \int_{-1}^{1} V(\xi)\, d\xi \quad \text{(the `flux')}, \\ C &= \frac{1}{\pi}\ln\left(\frac{2\pi s}{d}\right) - \frac{i}{n_2\bar{d}}. \end{aligned}\right\} \tag{3.6}$$

The solution of the singular integral equation (3.5) which possesses at most integrable singularities at the end-points $\xi = \pm 1$ is

$$V(\xi) = \frac{1}{\pi^2(1-\xi^2)^{\frac{1}{2}}}\left\{ ⨍_{-1}^{1} \frac{(1-\mu^2)^{\frac{1}{2}}}{\mu-\xi}\frac{dg}{d\mu}\, d\mu - \frac{1}{\ln 2}\int_{-1}^{1}\frac{g(\mu)}{(1-\mu^2)^{\frac{1}{2}}}\, d\mu\right\}, \tag{3.7}$$

(Carrier, Krook & Pearson 1966, p. 428), the first integral on the right-hand side being a principal value. Substituting for $g(\mu)$ from (3.6), and making use of the generating function for Bessel functions J_n (Abramowitz & Stegun 1964, p. 361) we find:

$$V(\xi) = \frac{1}{\bar{s}(1-\xi^2)^{\frac{1}{2}}}\left\{\frac{(1+C\bar{s}Q)}{\ln 2} - \frac{\alpha J_0(\kappa s)}{2\ln 2} - \frac{i\alpha\kappa s}{2}\left[\xi J_0(\kappa s) + iJ_1(\kappa s) - 2(1-\xi^2)^{\frac{1}{2}}\sum_{n=1}^{\infty} i^n \sin n\vartheta\, J_n(\kappa s)\right]\right\}, \tag{3.8}$$

where

$$\vartheta = \cos^{-1}\xi. \tag{3.9}$$

This formal expression for $V(\xi)$ involves the unknown parameters α, Q. The Kutta condition requires that $V(\xi)$ remains finite as $\xi \to -1$, and thence that the term in the curly brackets of (3.8) vanishes as $\xi \to -1$, which gives

$$\alpha = \frac{-2(1+C\bar{s}Q)}{\{i\kappa s \ln 2\,(J_0(\kappa s) - iJ_1(\kappa s)) - J_0(\kappa s)\}}. \tag{3.10}$$

Substituting this result into equation (3.8) and integrating with respect to ξ over $(-1, 1)$ then yields, from (3.6),

$$Q = \frac{-in_2(d/s)}{\left\{1 - \dfrac{in_2\bar{d}}{\pi}\ln\left(\dfrac{d}{\pi s}\right) + \dfrac{n_2 d}{\pi\kappa s}\dfrac{J_0(\kappa s)}{[J_0(\kappa s) - iJ_1(\kappa s)]}\right\}}. \tag{3.11}$$

This completes the formal analysis. Equations (3.8), (3.10), (3.11) determine the acoustic field in $X_2 \gtrless 0$ by means of equations (2.21), (2.23).

When the condition (3.1) is fulfilled the reflected and transmitted fields reduce to specularly reflected and transmitted waves at large distances from the screen. These are determined from (2.21), (2.23) by making use of the integral representation of the Hankel function

$$H_0^{(1)}[\Gamma((X_1 - Y_1 - N\bar{d})^2 + X_2^2)^{\frac{1}{2}}] = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\exp[i\{\lambda(X_1 - Y_1 - N\bar{d}) + |X_2|\,(\Gamma^2 - \lambda^2)^{\frac{1}{2}}\}]\,d\lambda}{(\Gamma^2 - \lambda^2)^{\frac{1}{2}}}, \tag{3.12}$$

and the identity

$$\frac{1}{2l}\sum_{N=-\infty}^{\infty} e^{iN\pi x/l} = \sum_{N=-\infty}^{\infty} \delta(x - 2Nl), \tag{3.13}$$

(Lighthill 1958, §5.4). When $X_2 \to +\infty$ it follows from (2.21) that

$$\psi = e^{i(n_1 X_1 - n_2 X_2)} + R e^{i(n_1 X_1 + n_2 X_2)} + \ldots, \tag{3.14}$$

where the terms omitted are exponentially small when the Helmholtz number is small, and the reflexion coefficient R is given by

$$R = 1 - iQs/n_2 d. \tag{3.15}$$

Similarly, the transmitted field reduces to the plane wave $T e^{i(n_1 X_1 - n_2 X_2)}$ at large distances, where the transmission coefficient

$$T = iQs/n_2 d. \tag{3.16}$$

These expressions for the reflexion and transmission coefficients reduce to those given by Lamb (1932, §306) in the corresponding limit of low Helmholtz number, provided that the terms involving Bessel functions – which characterize vortex shedding – are discarded, and $\bar{d}$ is set equal to d (no mean flow).

4. Discussion and conclusion

In order to interpret these analytical results they must be expressed in terms of the original co-ordinate system (x_1, x_2, x_3), and the precise form of the incident wave Φ_I must be specified. It is convenient to characterize the direction of propagation of this wave in terms of the normal to the wave front, although this does not, of course, coincide with the direction of acoustic energy propagation in the moving medium.

Let the wave normal be determined by the polar direction (θ, ϕ), $(0 \leqslant \theta \leqslant \frac{1}{2}\pi$, $0 \leqslant \phi \leqslant 2\pi)$, where θ is the angle of incidence measured from the positive x_2 direction normal to the screen (see figure 1), and ϕ is the azimuthal angle measured from the positive direction of the x_1 axis. The incident wave Φ_I, which is a solution of the convected wave equation (2.1), may then be taken as

$$\Phi_I = \exp\left[\frac{ik}{(1+M\sin\theta\,\cos\phi)}(x_1\sin\theta\cos\phi - x_2\cos\theta + x_3\sin\theta\sin\phi)\right]. \tag{4.1}$$

The angular frequency ω is measured in a frame which is fixed relative to the screen, and the factor $(1+M\sin\theta\,\cos\phi)$ in (4.1) accounts for the Doppler shift in the wavelength produced by the mean flow. Reference to equations (3.11) and (3.15), (3.16) reveals that only the component n_2 of the wavenumber $\mathbf{n}$ defining the Prandtl–Glauert potential ψ is involved in the specification of the interaction of the sound with the screen, and we have from (2.6), (4.1)

$$n_2 = \frac{k\cos\theta}{1+M\sin\theta\,\cos\phi}. \tag{4.2}$$

The *open-area* ratio of the screen is given by

$$\sigma = 2s/d. \tag{4.3}$$

Define real valued functions $\gamma(\sigma, M, \kappa s)$, $\delta(\sigma, M, \kappa s)$ of σ, the mean flow Mach number M, and the *reduced frequency* $\kappa s \equiv \omega s/U_c$, by means of

$$\gamma = \frac{2M_c\cos\theta J_0(\kappa s)^2}{\pi\sigma(1-M^2)^{\frac{1}{2}}(1+M\sin\theta\,\cos\phi)\,[J_0(\kappa s)^2+J_1(\kappa s)^2]}, \tag{4.4}$$

$$\delta = \frac{-2M_c\cos\theta}{\pi\sigma(1-M^2)^{\frac{1}{2}}(1+M\sin\theta\,\cos\phi)}\left\{\kappa s\ln\left(\frac{2}{\pi\sigma}\right) - \frac{J_0(\kappa s)\,J_1(\kappa s)}{[J_0(\kappa s)^2+J_1(\kappa s)^2]}\right\}. \tag{4.5}$$

The reflexion and transmission coefficients defined respectively by (3.15), (3.16) may now be expressed in the form:

$$T = 1-R = 1/\{1+\gamma+i\delta\}. \tag{4.6}$$

These may be used to determine the acoustic energy transmitted and reflected by the screen. To do this recall that the mean acoustic power flux in the i direction is equal to $\langle\rho v_i h\rangle$, where v_i is the *total* velocity in the i direction, ρ, h are respectively the density and total enthalpy, and the angle brackets denote an average over a wave period (Landau & Lifshitz 1959, §§ 6 and 64). In the present case $h = \mathrm{Re}\{i\omega\Phi\, e^{-i\omega t}\}$, so that if Π_I is the acoustic power incident on the screen, and Π_s is the total reflected and transmitted acoustic powers, it follows that, with due account taken of convection by the mean flow,

$$\Pi_s/\Pi_I = |R|^2+|T|^2$$
$$\equiv 1-\frac{2\gamma}{\{(1+\gamma)^2+\delta^2\}}. \tag{4.7}$$

Since $\gamma > 0$, it is apparent that $\Pi_s \leqslant \Pi_I$, i.e., that acoustic energy is dissipated during the interaction with the screen. Note that $\gamma \to 0$ as the vorticity convection Mach number $M_c \to 0$, and acoustic energy cannot therefore be dissipated when the shed vorticity is not carried away by the mean flow. Also, if the Kutta condition had not been applied, so that Φ is continuous through the slits, with the constant α of (2.3) identically zero, but $M \neq 0$, all of the terms involving the Bessel functions would be absent from our final expressions, and acoustic energy would be conserved. The energy which is lost at the screen reappears as the essentially incompressible, hydrodynamic energy of the shed vorticity. Howe (1979*b*, *c*) has shown that the rate of dissipation is equal to the rate of working of the Reynolds stresses in the rate of strain field of the sound.

In considering the implications of these conclusions it must be recognized that implicit in the vortex-sheet-wake model is the assumption that the Reynolds number based on the slit scale $2s$ is large. Also, in practice the mean flow will be separated from the screen by boundary layers whose widths are generally large compared with $2s$. It may therefore be conjectured that the vorticity convection velocity U_c should be assigned a value which is somewhat smaller than that of the free stream. The case in which the mean boundary-layer flow is turbulent is of particular importance. The magnitude of the vorticity convection velocity is likely to be related to the mean velocity in the viscous region close to the wall, which is ultimately responsible for the generation of the shed vorticity, and it is probable that we shall not be substantially in error if U_c is identified with the mean velocity at the outer edge of the viscous sublayer. This implies that

$$U_c \simeq 5v_* \simeq 0{\cdot}2U, \tag{4.8}$$

where $v_* \simeq 0{\cdot}04U$ is the friction velocity (Hinze 1975, p. 615); in the following discussion we shall tentatively assume that $M_c = 0{\cdot}2M$. It might be argued that, since the validity of the diffraction analysis implicitly requires the slit width $2s$ to be large compared with the plate thickness, it would be more reasonable to take the convection velocity to be equal to the mean velocity at the inner edge of the fully-turbulent region of the boundary-layer flow, i.e., $U_c \simeq 0{\cdot}6U$. This would certainly be appropriate for flow over a blunt trailing edge, where the centre-line velocity accelerates from zero to about $0{\cdot}6U$ in a distance of the order of the trailing-edge thickness. For a screen of small open-area ratio, however, it seems unlikely that a significant acceleration of the mean flow is possible in the slits. Because of this ambiguity in the precise value of M_c it is advisable, where possible, to normalize numerical results with respect to the vorticity convection velocity rather than that of the free stream. Actually, it often happens that the mean flow Mach number is sufficiently small that it may be neglected in expressions (4.4), (4.5) for γ, δ; the vorticity convection Mach number is then the only component of the mean flow which need be retained in the formula for the dissipation. In this case M_c may be interpreted either as the uniform convection velocity in the idealized problem, or as the vorticity convection velocity in the real problem.

Let us consider first the case in which there is no mean flow ($M = 0$). The condition (3.1) requires the Helmholtz number of the sound to be small, and table 1 gives the variations of $|T|$, $|R|$ for $0 \leqslant kd \leqslant 0{\cdot}5$ in the case of normal incidence ($\theta = 0$) and a 5 % open area ratio ($\sigma = 0{\cdot}05$).

kd	$\|T\|$	$\|R\|$
0	1·0	0
0·1	0·9967	0·0807
0·2	0·9871	0·1599
0·3	0·9717	0·2361
0·4	0·9513	0·3082
0·5	0·9269	0·3753

TABLE 1

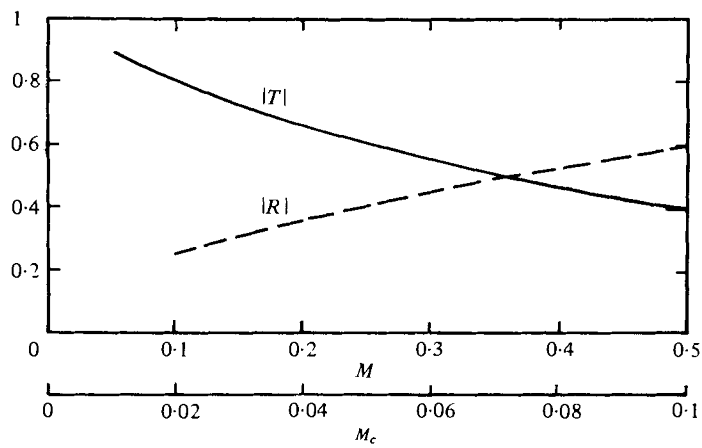

FIGURE 2. Variation of $|R|$, $|T|$ ($0 \leqslant kd \leqslant 0{\cdot}5$) with the mean Mach number M and the vorticity convection Mach number M_c when it is assumed that $M_c \equiv 0{\cdot}2M$. The curves illustrate the case of normal incidence ($\theta = 0°$) and an open area ratio of 5 %.

Acoustic energy is conserved, with the corresponding entries of table 1 satisfying $|T|^2 + |R|^2 = 1$. For $kd \leqslant 0{\cdot}5$, more than 85 % of the incident sound power is transmitted through the screen – an observation due originally to Rayleigh (1897). When there exists a subsonic grazing flow ($M \neq 0$), but in the *absence* of vortex shedding, the results of table 1 continue to be valid provided that the first column is taken to represent $kd/(1-M^2)^{\frac{1}{2}}$; as before, acoustic energy is conserved.

The dependence of $|T|$, $|R|$ on the Helmholtz number kd when vortex shedding is taken into account is given by equations (4.4)–(4.6). For an open area ratio of 5 % and at normal incidence ($\theta = 0$), these equations show that $|T|$ is essentially independent of kd when the eddy convection Mach number M_c exceeds 0·01 and $|R|$ is independent of kd for $M_c \gtrsim 0{\cdot}02$ (corresponding respectively to $M \gtrsim 0{\cdot}05$, 0·1, if the relation (4·8) is assumed to hold). The variations of $|T|$, $|R|$ with M, M_c over this range of reduced frequencies are illustrated in figure 2. Acoustic energy is dissipated at the screen ($|R|^2 + |T|^2 < 1$), and the vortex shedding is seen to modify considerably the fraction of the acoustic energy transmitted by the screen. For example, at $M = 0{\cdot}35$ ($M_c = 0{\cdot}07$) less than about 25 % of the incident acoustic energy is transmitted, and 50 % is absorbed by the vorticity.

The upper limit $kd = 0{\cdot}5$, say, of the anticipated range of validity of the present theory, imposes a corresponding upper limit on the reduced frequency κs, namely

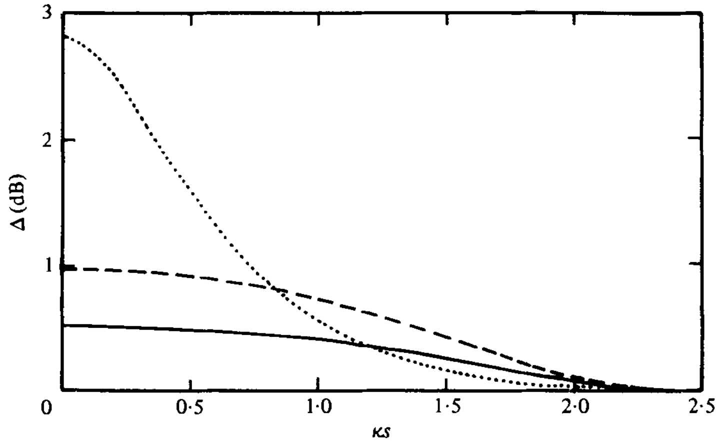

FIGURE 3. The dependence of the acoustic attenuation Δ dB on the reduced frequency κs, for fixed convection Mach number $M_c = 0{\cdot}01$ and $\theta = 0°$; ..., $\sigma = 0{\cdot}01$; – – –, $\sigma = 0{\cdot}05$; ——, $\sigma = 0{\cdot}1$.

$$\kappa s \equiv (kd)\,\sigma/2M_c \lesssim \sigma/4M_c. \tag{4.9}$$

In applications $\sigma \lesssim 0{\cdot}1$, and except for very small vorticity convection Mach numbers M_c, only moderate values of the reduced frequency are encompassed by the analysis.

The dependence of the dissipation on the reduced frequency is conveniently measured on a conventional logarithmic scale by

$$\Delta = -10 \times \log_{10}\{\Pi_s/\Pi_I\}. \tag{4.10}$$

For $M_c = 0{\cdot}01$, $\sigma = 0{\cdot}1$, (4.9) indicates that the relevant values of κs lie between zero and 2.5. Figure 3 shows the variation of the attenuation Δ over this range of κs, for $\sigma = 0{\cdot}01, 0{\cdot}05, 0{\cdot}1$, and for $\theta = 0$, $M_c = 0{\cdot}01$. In all cases Δ vanishes when $\kappa s \approx 2{\cdot}4$, at the first zero of the Bessel function $J_0(\kappa s)$ where, according to (4.4), γ also vanishes. Equation (3.11) shows that this is a situation in which the flux Q coincides in value with that it would have in the absence of vortex shedding, although $\alpha \neq 0$. In other words, the potential difference across the slits produced by the shed vorticity vanishes as a consequence of its equal and opposite successive interactions with the leading and trailing edges of the slits.

It is, perhaps, surprising that for $\kappa s \leqslant 0{\cdot}75$ the greatest attenuation occurs at the smallest open area ratio of $\sigma = 0{\cdot}01$. This conclusion, however, is strongly dependent on the flow Mach number. Figure 4 illustrates the dependence of Δ on M, M_c for $\kappa s \to 0$, which is probably the only case of interest in applications. It is evident that, although the greatest attenuation is provided by the smallest open area ratio $\sigma = 0{\cdot}01$ for sufficiently small Mach number, the situation is reversed when the mean flow velocity increases. Each of the curves of figure 4 attains a peak value of just over 3 dB, at $M_c = \frac{1}{2}\pi\sigma$, a criterion which may be of interest in design-practice in fixing the appropriate value of σ.

At very small Mach numbers equations (4.4), (4.7) indicate that, for fixed κs, the attenuation varies linearly with M_c, a prediction which is in agreement with experimental results of Ingard & Ising (1967). Howe (1979*c*) has given a general argument which shows that at low Mach numbers the net attenuation Π, say, due to the

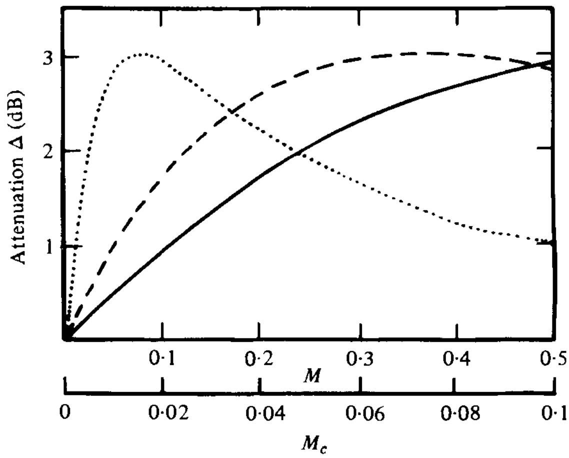

FIGURE 4. Variation of Δ with M, M_c ($M_c \equiv 0{\cdot}2M$) for $\kappa s = 0$ and for normal incidence ($\theta = 0°$); . . ., $\sigma = 0{\cdot}01$; – – –, $\sigma = 0{\cdot}05$; ——, $\sigma = 0{\cdot}1$.

vortex shedding mechanism discussed in this paper, together with that arising from the nonlinear 'jetting' of fluid in the slits, is given by

$$\Pi = \rho_0 \int \boldsymbol{\omega} \wedge \mathbf{v} . \mathbf{u} \, d^3\mathbf{x}. \tag{4.11}$$

In this result $\boldsymbol{\omega}$ is the vorticity, $\mathbf{v}$ the velocity, and $\mathbf{u}$ is the acoustic particle velocity (which is defined to be the total unsteady component of velocity less the incompressive component induced by the vorticity distribution $\boldsymbol{\omega}$ in accordance with the Biot–Savart law, with due account taken of the presence of rigid surfaces). In the absence of mean flow the integrand in (4.11) is third order in the acoustic particle velocity, and the resulting attenuation characterises that associated with nonlinear jetting. When U is non-zero, $\mathbf{v}$ has a component proportional to U_c and the integrand becomes quadratic in the acoustic amplitude. With decreasing mean flow velocity a stage is ultimately reached at which these two dissipative components are of comparable magnitude. The value of M_c at which this occurs depends, of course, on the acoustic amplitude. Some idea of the orders of magnitude involved is obtained by noting that nonlinearity is important when $u \sim U_c$. For a sound pressure level of $\mathcal{N}$ dB (re $0{\cdot}002\,\mu$bar), this implies that

$$M_c \sim 10^{-\{(194-\mathcal{N})/20\}}. \tag{4.12}$$

When $\mathcal{N} \sim 120$ dB, which is typical of the sound pressure level in a gas heat exchanger in the *presence* of the perforated plates, it appears that nonlinear dissipative processes will be comparable with the vortex shedding mechanism discussed in this paper only for $M_c \lesssim 10^{-4}$. In air this corresponds to a convection velocity which does not exceed about $4\,\mathrm{cm\,s^{-1}}$, and it may be concluded that in practice acoustic nonlinearity is probably of no significance.

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